<u>Question 1</u> (15 Marks) Marks

(a) Integrate with respect to x:

(i) $x^2 - \sqrt{x}$ **2**

(ii) $\sin 8x$ **2**

(iii) $\dfrac{6}{1-3x}$ **2**

(b) Evaluate:

(i) $\int_0^3 e^{2x+1}\, dx$ **2**

(ii) $\int_0^{\pi} \cos\left(\frac{1}{4}x\right) dx$ **2**

(iii) $\int_1^4 \dfrac{2x^2-3}{x}\, dx$ **2**

(c) (i) Evaluate $\sum_{r=1}^{5} r^2$ **1**

(ii) Find the sum of the first 200 terms of the arithmetic series: 3 + 7 + 11 + 15 + … **2**

<u>Question 2</u> (15 Marks) Marks

(a) Find:

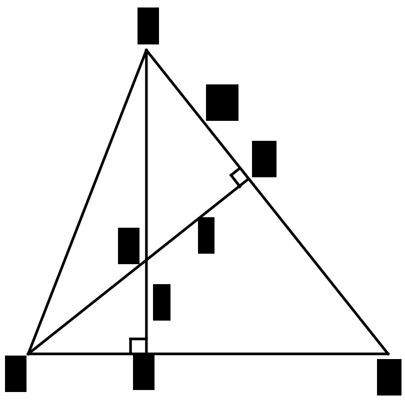
(i) $\int (3x-2)^8\, dx$ **2**

(ii) $\int \dfrac{e^{3x}}{5+e^{3x}}\, dx$ **2**

(b) (i) Sketch the curve $y = 6x - x^2$. Clearly show its intercepts with the coordinate axes. **2**

(ii) Find the area bounded by the curve $y = 6x - x^2$ and the x-axis. **3**

(c) In ΔABC, $AP \perp BC$ and $BQ \perp AC$.



(i) Prove that $\Delta AQR \parallel \Delta BPR$. **3**

(ii) If $AQ = 12$, $RQ = 9$ and $RP = 6$, find the area of ΔABR. **3**

Question 3 (15 Marks) Marks

(a) The area bounded by the curve $y=\sqrt{4-x}$ and the coordinate axes is rotated one revolution about the y-axis. Find the volume of the solid formed. **3**

(b) (i) Show that the curves $y=\cos x$ and $y=\sin 2x$ meet at a point where $x=\frac{\pi}{6}$. **2**

(ii) On the same set of axes draw a neat <u>half page</u> diagram of the curves $y=\cos x$ and $y=\sin 2x$ for $0\le x\le\frac{\pi}{2}$ clearly showing their points of intersection. **2**

(iii) Find the area bounded by the above curves for $0\le x\le\frac{\pi}{2}$. **3**

(c) Mr. Green sets up a fund to pay for a future holiday by paying \$500 into the fund at the start of each <u>month</u>. The fund pays 0.5% interest at the end of each <u>month</u> on the balance of money in the fund.

(i) Find the value of the fund at the end of the first year. **2**

(ii) How long must Mr. Green pay into the fund if he needs \$20000 for his holiday. Give your answer to the nearest month. **3**

Question 4 (15 Marks) Marks

(a) An artist decides to make a design showing a sequence of concentric rope circles. The inner most circle has a radius of 40 cm and each extra circle has a radius that is 10 cm larger than the radius of the previous circle.

(i) Find the amount of rope used to make the tenth circle. Give your answer to the nearest <u>metre</u>. **2**

(ii) If the artist has 2 km of rope, how many complete circles will be in the design? **3**

(b) (i) Given that $f(x)=\ln(\cos 2x)$, find $f'(x)$. **1**

(ii) Prove that the equation of the tangent to $y=\tan 2x$ at the point $P\left(\frac{\pi}{8},1\right)$ has the equation $y=4x+1-\frac{\pi}{2}$. **3**

(iii) Find the coordinates of Q, the point where the tangent at P crosses the x-axis. **1**

(iv) On a neat <u>half page</u> diagram, sketch the curve $y=\tan 2x$ for $0\le x\le\frac{\pi}{4}$ and the tangent at P. **2**

(v) Find the area bounded by the curve $y=\tan 2x$, the tangent at P and the x-axis. Express your answer in the form $a+b\ln(2)$, where a and b are rational numbers. **3**

This is the end of the Examination Paper

JRAHS Year 12 Term 1 Mathematics (2 Unit) Assessment – 2006

SOLUTIONS

Question 1 (15 Marks)

Solution	Marking criteria	Marks
(a) Integrate with respect to x:		
(i) $\frac{1}{3}x^3 - \frac{2}{3}x\sqrt{x} + c$	each part = 1 mark - 1 for no “c” only once in part (a)	**2**
(ii) $-\frac{1}{8}\cos 8x + c$	$\cos 8x$ = 1 mark -1/8 …… = 1 mark	**2**
(iii) $-2\ln(1-3x)+c$ or $-2\ln\lvert 1-3x\rvert + c$	$\ln(1-3x)$ = 1 mark -2 ….. = 1 mark	**2**
(b) Evaluate:		
(i) $\int_0^3 e^{2x+1}\,dx = \left[\frac{1}{2}e^{2x+1}\right]_0^3$ $= \frac{1}{2}\left(e^7 - e^1\right)$	primitive = 1 mark evaluation = 1 mark	**2**
(ii) $\int_0^{\pi} \cos\left(\frac{1}{4}x\right)dx = \left[4\sin\left(\frac{1}{4}x\right)\right]_0^{\pi}$ $= 4\left(\sin\left(\frac{\pi}{4}\right) - \sin 0\right)$ $= 4\times\frac{1}{\sqrt{2}} - 0$ $= 2\sqrt{2}$	primitive = 1 mark evaluation = 1 mark	**2**
(iii) $\int_1^4 \frac{2x^2-3}{x}\,dx = \int_1^4 \left(2x - \frac{3}{x}\right)dx$ $= \left[x^2 - 3\ln x\right]_1^4$ $= (16 - 3\ln 4) - (1 - 3\ln 1)$ $= 15 - 3\ln 4$	primitive = 1 mark evaluation = 1 mark	**2**
(c) (i) $\sum_{r=1}^{5} r^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$ $= 55$	accept correct answer with no working	**1**
(ii) $n = 200, \quad a = 3, \quad d = 4$ $S_{200} = \frac{200}{2}\{2\times 3 + 199\times 4\}$ $= 80200$	sub. into correct formula = 1 mark correct sum = 1 mark	**2**

Question 2 (15 Marks) Marks

(a) Find:

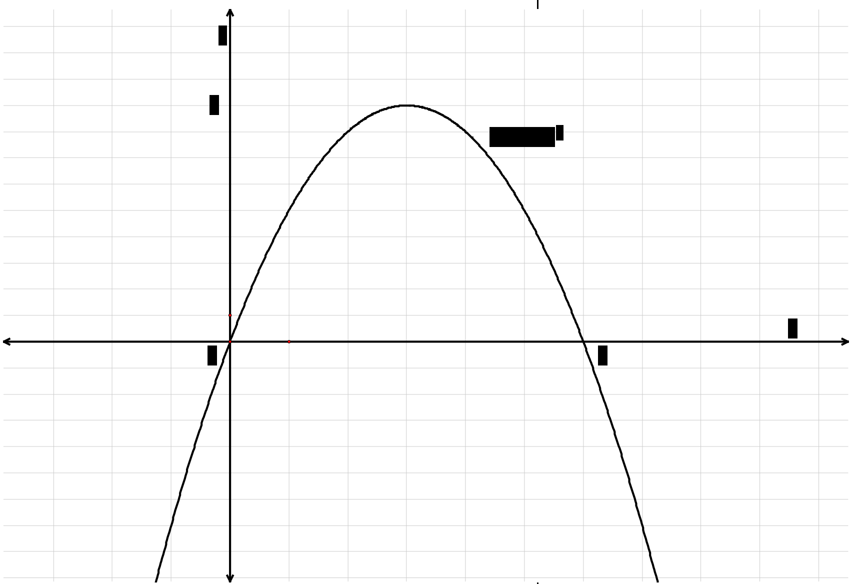
(i) $\int (3x-2)^8\,dx = \frac{1}{27}(3x-2)^9 + c$

$(3x-2)^9$ = 1 mark
no penalty for missing "c" **2**

(ii) $\int \frac{e^{3x}}{5+e^{3x}}\,dx = \frac{1}{3}\ln\left(5+e^{3x}\right) + c$

$\ln\left(5+e^{3x}\right)$ = 1 mark
no penalty for missing "c" **2**

(b) (i) **2**



both intercepts = 1 mark
shape = 1 mark

(ii)
$$A = \int_0^6 \left(6x - x^2\right)dx$$
$$= \left[3x^2 - \tfrac{1}{3}x^3\right]_0^6$$
$$= \left(3\times 6^2 - \tfrac{1}{3}\times 6^3\right) - (0-0)$$
$$= 36$$
$$\therefore\ \text{Area} = 36\,u^2$$

correct integral = 1 mark **3**

integration = 1 mark

evaluation = 1 mark

max. 2/3 if no units

(c) In ΔABC, $AP \perp BC$ and $BQ \perp AC$.

(i)

In $\Delta AQR \,|||\, \Delta BPR$ **3**

$A\hat{Q}R = B\hat{P}R \quad (\text{both } 90^\circ)$

angle pairs (= 1 mark each) = 2 marks

$A\hat{R}Q = B\hat{R}P \quad \begin{pmatrix}\text{vertically opposite} \\ \text{angles are equal}\end{pmatrix}$

$\therefore \Delta A\hat{Q}R \,|||\, \Delta B\hat{P}R \quad (\text{equiangular})$

test = 1 mark

(ii)

$\text{Area } \Delta ABR = \dfrac{BR \times AQ}{2}$ **3**

$\dfrac{BP}{12} = \dfrac{6}{9} \quad \begin{pmatrix}\text{ratio of corresponding} \\ \text{sides in similar triangles} \\ \text{are equal}\end{pmatrix}$

calculating 2 needed lengths (= 1 mark each) = 2 marks

several pairs of sides and various triangle combinations can be used to find the required area

$BP = 8$

$BR^2 = 8^2 + 6^2 \; (\text{Pythagoras' Theorem})$

$BR = 10$

$\text{Area} = \dfrac{10 \times 12}{2} u^2$

$= 60\, u^2$

area = 1 mark

Question 3 (15 Marks) Marks

(a) $V = \pi \int_0^2 x^2 \, dy$ where $y = \sqrt{4 - x}$ **3**

$y^2 = 4 - x$

$x = 4 - y^2$

$$\begin{aligned} V &= \pi \int_0^2 \left(4 - y^2\right)^2 \, dy \\ &= \pi \int_0^2 \left(16 - 8y^2 + y^4\right) dy \\ &= \pi \left[16y - \tfrac{8}{3}y^3 + \tfrac{1}{5}y^5\right]_0^2 \\ &= \pi \left\{\left(16 \times 2 - \tfrac{8}{3} \times 8 + \tfrac{1}{5} \times 32\right) - 0\right\} \\ &= \tfrac{256\pi}{15} \end{aligned}$$

$\therefore \ Volume = \frac{256\pi}{15} u^3$

integral = 1 mark

wrong intergrand = max 1/3

integration = 1 mark

evaluation = 1 mark

no penalty for missing units

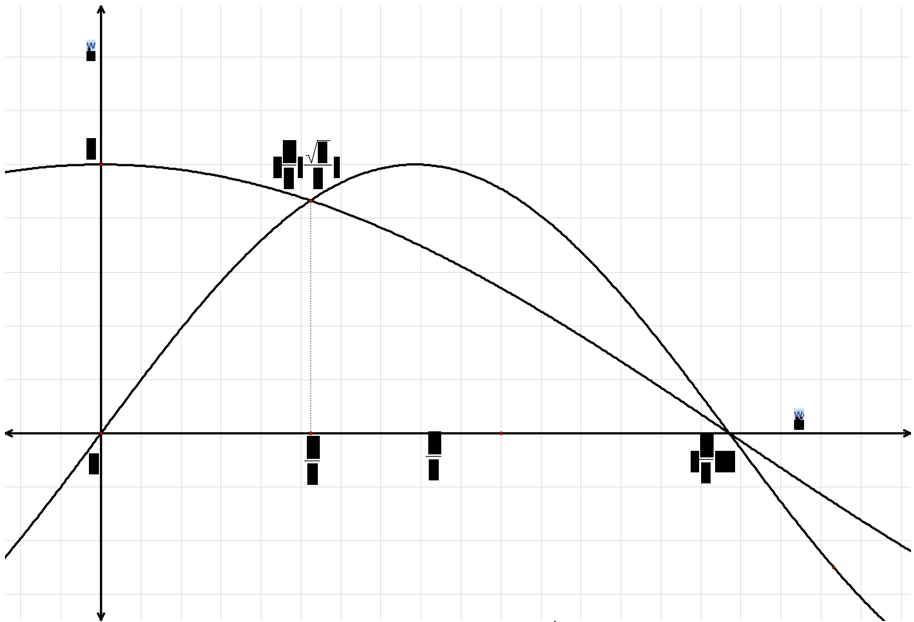
(b) (i) when $x = \frac{\pi}{6}$ **2**

$$\begin{aligned} \cos x &= \cos \tfrac{\pi}{6} \\ &= \tfrac{\sqrt{3}}{2} \\ \sin 2x &= \sin \tfrac{2\pi}{6} \\ &= \sin \tfrac{\pi}{3} \\ &= \tfrac{\sqrt{3}}{2} \end{aligned}$$

$\therefore$ both curves meet at $\left(\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right)$

show that each y value = $\frac{\sqrt{3}}{2}$
(= 1 mark each) = 2 marks

(ii) **2**



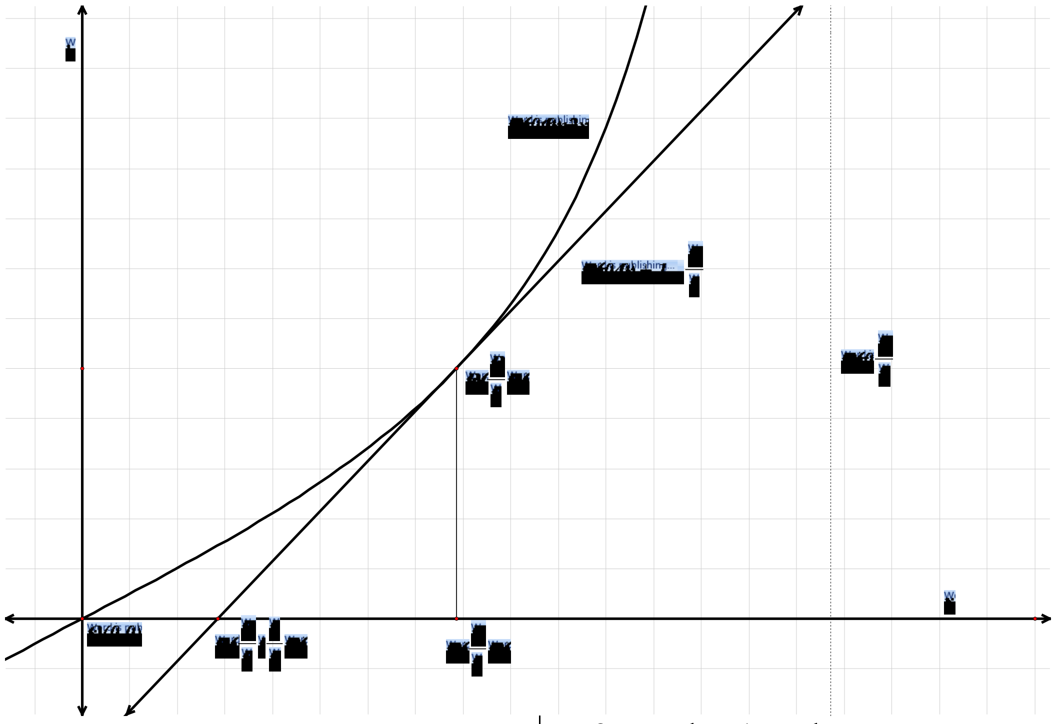
1 mark for each curve
no penalty if domain extends beyond $\frac{\pi}{2}$
no penalty for missing intersection point

		Solution	Marking criteria	Marks
	(iii)	$A=\int_{\pi/6}^{\pi/2}\left(\sin 2x-\cos x\right)dx$	integral = 1 mark	**3**
		$=\left[-\tfrac{1}{2}\cos 2x-\sin x\right]_{\pi/6}^{\pi/2}$	integration = 1 mark	
		$=\left(-\tfrac{1}{2}\cos\pi-\sin\left(\tfrac{\pi}{2}\right)\right)-\left(-\tfrac{1}{2}\cos\left(\tfrac{2\pi}{6}\right)-\sin\left(\tfrac{\pi}{6}\right)\right)$		
		$=\tfrac{1}{4}$	evaluation = 1 mark	
		$\therefore$ Area $=\tfrac{1}{4}u^2$	no penalty for missing units	
(c)	(i)	Let value of fund at end of n[th] month = $\$A_n$		**2**
		$A_1=500\times 1.005$		
		$A_2=\left(A_1\times 1.005+500\right)\times 1.005$		
		$=500\left(1.005^2+1.005\right)$		
		$\vdots$		
		$\vdots$		
		$A_{12}=500\left(1.005^{12}+1.005^{11}+...+1.005\right)$	correct expression = 1 mark	
		$=500\times 1.005\dfrac{\left(1.005^{12}-1\right)}{1.005-1}$		
		$=6167.78$	evaluation of sum = 1 mark	
		Value $=\$6167.78$		
	(ii)	$A_n=500\left(1.005^n+1.005^{n-1}+...+1.005\right)$	correct general expression = 1 mark	**3**
		$=500\times 1.005\dfrac{\left(1.005^n-1\right)}{0.005}$		
		$=100500\left(1.005^n-1\right)$		
		$\therefore\ 20000=100500\left(1.005^n-1\right)$		
		$1.005^n=\tfrac{241}{201}$	simplified equation = 1 mark	
		$n\ln(1.005)=\ln\left(\tfrac{241}{201}\right)$		
		$n=\dfrac{\ln\left(\frac{241}{201}\right)}{\ln(1.005)}$		
		≈ 36.39	solve equation = 1 mark (by trial and error or logs)	
		no. months $=36$	accept 37 months	

Question 4 (15 Marks) Marks

(a) (i) circumferences: $\{80\pi, 100\pi, 120\pi, ...\}$ $a = 80\pi, d = 20\pi$ $T_{10} = 80\pi + 9\times 20\pi$ $= 260\pi$ Length $= 260\pi\ cm$ $\approx 816.81\, cm$ $= 8m$ (to nearest metre)	indication of correct "a" and "d" = 1 mark evaluate length = 1 mark (using their "a" and "d") correct decimal approx. = 1 mark	**2**
(ii) $S_n = \frac{n}{2}\{2\times 80\pi + (n-1)\times 20\pi\}$ when $S_n = 200000$ $\frac{n}{2}\{2\times 80\pi + (n-1)\times 20\pi\} = 200000$ $\pi n^2 + 7\pi n - 20000 = 0$ $n = \frac{-7\pi \pm \sqrt{80049\pi}}{2\pi}$ $n \approx -83.2$ or 76.3 but $n > 0$ $\therefore$ no. complete circles = 76	sum formula with substitution = 1 mark simplified quadratic equation = 1 mark correct approximation = 1 mark	**3**
(b) (i) $f(x) = \ln(\cos 2x)$ $f'(x) = \frac{-2\sin 2x}{\cos 2x}$ $= -2\tan 2x$		**1**
(ii) $y = \tan 2x$ $y' = 2\sec^2 2x$ when $x = \frac{\pi}{8}$ $y' = 2\sec^2\left(\frac{\pi}{4}\right)$ $= 2\left(\sqrt{2}\right)^2$ $= 4$ tangent is: $y - 1 = 4\left(x - \frac{\pi}{8}\right)$ $y - 1 = 4x - \frac{\pi}{2}$ $y = 4x + 1 - \frac{\pi}{2}$	derivative = 1 mark gradient = 1 mark tangent equation = 1 mark	**3**
(iii) at Q, $y = 0$ $\therefore\ 4x + 1 - \frac{\pi}{2} = 0$ $x = \frac{\pi}{8} - \frac{1}{4}$ Q is $\left(\frac{\pi}{8} - \frac{1}{4}, 0\right)$		**1**

(iv) **2**



tan2x graph = 1 mark

graph of tangent line = 1 mark

no penalty if domain extends beyond $\frac{\pi}{4}$

(v) **3**

$$A = \int_0^{\pi/8} \tan 2x \, dx - \frac{QR \times PR}{2}$$

$$= \int_0^{\pi/8} \tan 2x \, dx - \frac{\frac{1}{4} \times 1}{2}$$

$$= \left[-\tfrac{1}{2} \ln(\cos 2x) \right]_0^{\pi/8} - \tfrac{1}{8}$$

$$= -\tfrac{1}{2} \left\{ \ln\left(\cos \tfrac{\pi}{4}\right) - \ln(\cos 0) \right\} - \tfrac{1}{8}$$

$$= -\tfrac{1}{2} \left\{ \ln\left(\tfrac{1}{\sqrt{2}}\right) - \ln(1) \right\} - \tfrac{1}{8}$$

$$= -\tfrac{1}{2} \left\{ -\tfrac{1}{2} \ln 2 - 0 \right\} - \tfrac{1}{8}$$

$$= -\tfrac{1}{8} + \tfrac{1}{4} \ln 2$$

$$\therefore \text{ Area} = \left(-\tfrac{1}{8} + \tfrac{1}{4} \ln 2 \right) u^2$$

area of triangle = 1 mark

$\left[-\tfrac{1}{2} \ln(\cos 2x) \right]_0^{\pi/8}$ = 1 mark

correct values for "a" and "b" = 1 mark

This is the end of the Examination Paper